Math 116 Section 04

Quiz 6 Name _____
August 05, 2005
Instructor: Charles Cuell Student Number _____

All solutions are to be presented on the paper in the space provided. The quiz is open book. You can discuss the problem with others and ask the TA questions.

(1) Consider the region bounded by $y = \dfrac{1}{x}$, $y = 0$, $x = 1$ and $x = 2$.
Find the volume obtained by:
(a) rotating the region about the x–axis.
Use the method of disks.

$$\begin{aligned} V &= \int_a^b A(x)\,dx \\ &= \int_1^2 \pi r^2\,dx \\ &= \pi \int_1^2 \frac{1}{x^2}\,dx \\ &= \left(-\pi \frac{1}{x}\right)\Big|_1^2 \\ &= -\pi\left(\frac{1}{2} - 1\right) \\ &= \frac{1}{2} \end{aligned}$$

(b) rotating the region about the y–axis.
Use the method of cylindrical shells.

$$\begin{aligned} V &= \int_a^b A(x)\,dx \\ &= \int_1^2 2\pi r h\,dx \\ &= \int_1^2 \pi x \frac{1}{x}\,dx \\ &= \int_1^2 \pi\,dx \\ &= \pi x\big|_1^2 \\ &= \pi(2-1) \\ &= \pi \end{aligned}$$

(2) Verify the mean value theorem for integrals for the function $f(x) = \sqrt{1-x}$ over the interval $[\,0\,,\,1\,]$.
The mean value theorem says that there is a $c \in [\,0\,,\,1\,]$ such that $f(c)(b-a) = \int_a^b f(x)\,dx$. Compute the integral first:

$$\begin{aligned} \int_0^1 \sqrt{1-x}\,dx &= -\frac{2}{3}(1-x)^{3/2}\Big|_0^1 \\ &= -\frac{2}{3}(0-1) \\ &= \frac{2}{3} \end{aligned}$$

Then, solve the equation $f(c)(1-0) = \frac{2}{3}$:

$$\begin{aligned} \sqrt{1-c}\,(1-0) &= \frac{2}{3} \\ \sqrt{1-c} &= \frac{2}{3} \\ 1-c &= \frac{4}{9} \\ c &= \frac{5}{9} \end{aligned}$$